Loss of Quantum Coherence and Positivity of Energy Density in Semiclassical Quantum Gravity

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In the semiclassical quantum gravity derived from the Wheeler-DeWitt equation, the energy density of a matter field loses quantum coherence due to the induced gauge potential from the parametric interaction with gravity in a non-static spacetime. It is further shown that the energy density takes only positive values and makes superposition principle hold true. By studying a minimal massive scalar field in a FRW spacetime background, we illustrate the positivity of energy density and obtain the classical Hamiltonian of a complex field from the energy density in coherent states.

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There has been a reviving interest in the back-reaction problem in black hole and wormhole physics. The complete resolution of the puzzle of back-reaction should be sought in quantum gravity, but at present there is not known any viable theory of quantum gravity, free from all problems. Though quantum gravity is not available at hand, semiclassical treatment of a gravity-matter system, quantized matter field and classical background spacetime, sheds light on some important aspects of quantum effects. For instance, the issue such as quantum interference or loss of quantum coherence can be treated in semiclassical gravity without relying on quantum gravity. In this semiclassical gravity there have been developed two typical methods: one is the traditional approach to semiclassical gravity [1] and the other is the so-called semiclassical quantum gravity approach [2]. In the traditional approach, one first quantizes the matter field on the fixed classical spacetime background, for instance, a la the functional Schrödinger equation. One then employs diverse methods to evaluate the expectation value of quantum stress-energy tensor and finally solves the semiclassical Einstein equation, $G_{\mu\nu} = 8\pi\langle\hat{T}_{\mu\nu}\rangle$. One readily sees that quantum interference predominates in the energy density expectation value and indeed leads to possible negative energy density [3].

On the other hand, in the semiclassical quantum gravity, one first quantizes both the geometry and matter field within the framework of canonical quantum gravity based on the Wheeler-DeWitt equation. From the Wheeler-DeWitt equation in (semi-)classical regions one derives the semiclassical quantum gravity: the Einstein-Hamilton-Jacobi equation or equivalently the semiclassical Einstein equation, $G_{\mu\nu} = 8\pi\langle\langle\hat{T}_{\mu\nu}\rangle\rangle$ and the time-dependent functional Schrödinger equation for the matter field. There is one noticeable difference from the traditional approach: the role of induced gauge potential [4–8]. The Wheeler-DeWitt equation for the gravity-matter system is analogous to the Schrödinger equation with zero energy for a molecular system: the matter fields play the role of electrons (fast particles) and the gravity that of nuclei (slow particles). In particular, it is observed that the off-diagonal elements of the Hamiltonian and the induced gauge potential cancel among themselves in the effective gravitational equation in a matrix form [5]. So one may expect the expectation values of quantum energy density to differ from each other in the two approaches to semiclassical gravity.

In this Rapid communication we study the effects of induced gauge potential of the gravity-matter system on the energy density within the framework of the semiclassical quantum gravity. It is found that the energy density for a superposed quantum state of matter field loses quantum coherence through the parametric interaction with gravity in a non-static spacetime. The loss of coherence in turn leads both to superposition principle for any exclusive set of quantum states and to the positivity of energy density. We compare these results with those from the traditional approach in which superposition principle does not hold true due to quantum interference among the quantum states and the energy density may take a negative value [3]. For this purpose we elaborate further the formalism developed in Refs. [6,8] to make the role of gauge potential be exhibited for the superposed quantum state. Through the study of a minimal massive scalar field in a FRW spacetime background, we also illustrate how classical matter Hamiltonian emerges from the decohered energy density.

Let us consider the Wheeler-DeWitt equation for a gravity-matter system

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$$\left[-\frac{\hbar^2}{2m_P^2}\nabla^2 - m_P^2 V(h_a) + \mathbf{H}\Big(\frac{\hbar}{i}\frac{\delta}{\delta\phi}, \phi; h_a\Big)\right]\Psi(h_a, \phi) = 0, \tag{1}$$

where $m_P = \frac{1}{\sqrt{G}}$ is the Planck mass, h_a and ϕ represent the superspace coordinates and the matter field, respectively. The semiclassical quantum gravity is obtained by applying the Born-Oppenheimer idea to separate the Wheeler-DeWitt equation into an effective gravitational field (heavy particle) equation and a time-dependent Schrödinger equation for the matter field (light particle). And then according to the de Broglie-Bohm interpretation the effective gravitational field equation reduces to the Einstein-Hamilton-Jacobi equation with quantum corrections. This scheme is valid in the (semi-)classical regions of superspace where the gravitational wave function oscillates and therefrom decohered (semi-)classical spacetime emerges.

We now wish to see how the matter field in the superposed quantum state loses its quantum coherence through the parametric interaction with gravity in a non-static spacetime. The matter field sector is assumed to have a well-defined Hilbert space by whose bases the wave functions can be expanded. We confine our attention to a complex wave function and study the evolution of quantum field along a single-branch of history, which includes the Vilenkin's tunneling wave function but excludes the Hartle-Hawking's no-boundary wave function [9]. Then the wave function can always be written as

$$\Psi(h_a, \phi) = \psi(h_a)|\Phi(\phi; h_a)\rangle, \tag{2}$$

$$|\Phi(\phi; h_a)\rangle = \sum_{n\in\mathcal{S}} c_n |\Phi_n(\phi; h_a)\rangle, \tag{3}$$

where $|\Phi\rangle$ has a unit norm and $\{\Phi_n\}$ forms an orthonormal basis of the Hilbert space. Any wave function that is superposed of more than two complex wave functions of the Wheeler-DeWitt equation can be rewritten as Eq. (2) through complex transformations in the Hilbert space. For this single-branch of history one is able to derive consistently the semiclassical quantum gravity without many fundamental conceptual problems mentioned in Ref. [2].

The quantum state (3) depends on the superspace as parameters, so a gauge potential is induced as the quantum state evolves on the superspace. The induced gauge potential of the matter field is divided into the diagonal and the off-diagonal part, $\mathbf{A}_D$ and $\mathbf{A}_O$, respectively:

$$\mathbf{A} = \langle\Phi|i\hbar\nabla|\Phi\rangle = \sum_{k,n\in\mathcal{S}} c_k^* c_n \langle\Phi_k|i\hbar\nabla|\Phi_n\rangle = \mathbf{A}_D + \mathbf{A}_O. \tag{4}$$

In order to obtain correctly the semiclassical Einstein equation with quantum back-reaction, it is necessary to treat the gauge potential appropriately. According to Ref. [5], Eq. (2) can be written as $\mathbf{U}^T(\phi; h_a)\cdot\mathbf{\Psi}(h_a)$, where $\mathbf{U}$ is the column vector consisted of $|\Phi_n\rangle$ and $\mathbf{\Psi}$ is the column vector consisted of $c_n\psi$. There it has been shown that the off-diagonal elements of the Hamiltonian and the induced gauge potential, $\mathbf{A}_O$, cancel among themselves in the effective gravitational equation (18) of Ref. [5]. The remaining energy density then consists of only diagonal elements. We can show these facts more directly by appropriately using the gauge potential but without relying on the matrix equation. The idea is to multiply the gravitational wave function by a phase factor from the diagonal part of the induced gauge potential and to compensate it by multiplying the quantum state with the phase factor of the opposite sign:

$$\Psi(h_a, \phi) = \left[e^{\frac{i}{\hbar}\int \mathbf{A}_{D,a}\cdot dh_a}\psi(h_a)\right] \times \left[e^{-\frac{i}{\hbar}\int \mathbf{A}_{D,a}\cdot dh_a}|\Phi(\phi; h_a)\rangle\right]. \tag{5}$$

The total wave function (2) is still invariant under the above gauge choice. For the gravitational wave function of the form

$$\psi(h_a) = F(h_a) e^{\frac{i}{\hbar}S(h_a)}, \tag{6}$$

the de Broglie-Bohm interpretation separates the real and imaginary parts of the Wheeler-DeWitt equation. The semiclassical Einstein equation comes from the real part

$$\begin{aligned}&\left[\frac{1}{2m_P^2}\Big(\nabla S\Big)^2 - m_P^2 V + \mathcal{H}_D + \mathcal{H}_O + \mathcal{H}_{vac.} - \frac{1}{m_P^2}\mathbf{A}_O\cdot\nabla S\right] \\ &+ \left[\frac{1}{2m_P^2}\mathbf{A}_D^2 - \frac{1}{m_P^2}\mathbf{A}\cdot\mathbf{A}_D - \frac{\hbar^2}{2m_P^2}\frac{\nabla^2 F}{F} + \frac{1}{2m_P^2}\langle\Phi|(i\hbar\nabla)^2|\Phi\rangle\right] = 0,\end{aligned} \tag{7}$$

where $\mathcal{H}_D$, $\mathcal{H}_O$, and $\mathcal{H}_{\rm vac.}$ are the diagonal, off-diagonal parts, and the vacuum energy of the quantum back-reaction:

$$\begin{aligned}\langle\Phi|\mathbf{H}|\Phi\rangle &= \sum_{k,n\in\mathcal{S}} c_k^* c_n \langle\Phi_k| : \mathbf{H} : |\Phi_n\rangle + \sum_{n\in\mathcal{S}} c_n^* c_n \langle\Phi_n|\mathbf{H} - : \mathbf{H} : |\Phi_n\rangle \\ &\equiv \mathcal{H}_D + \mathcal{H}_O + \mathcal{H}_{\rm vac.}. \end{aligned} \tag{8}$$

The vacuum energy will be absorbed into the cosmological constant and renormalize the potential, $V_{\rm ren.}$. The imaginary part can be integrated for F in terms of S and put into Eq. (7) in a self-consistent way [8].

On the other hand, by subtracting Eq. (7) from Eq. (1) after acting by $\langle\Phi|$ and by introducing a cosmological (WKB) time

$$\frac{\delta}{\delta\tau} = \frac{1}{m_P^2}\nabla S\cdot\nabla, \tag{9}$$

one derives the time-dependent Schrödinger equation for the matter field

$$i\hbar\frac{\delta}{\delta\tau}|\Phi(\tau)\rangle = \Bigg[\mathbf{H} + \mathbf{H}_{UQ} + \mathbf{H}_{NQ} + \mathcal{H}_{UC} + \mathcal{H}_{NC}\Bigg]|\Phi(\phi;h_a)\rangle, \tag{10}$$

where

$$\begin{aligned}\mathbf{H}_{UQ} &= -i\frac{\hbar}{m_P^2}\mathbf{A}\cdot\nabla - \frac{\hbar^2}{2m_P^2}\nabla^2, \\ \mathbf{H}_{NQ} &= -\frac{\hbar^2}{m_P^2}\frac{\nabla F}{F}\cdot\nabla \end{aligned} \tag{11}$$

are operator-valued quantum corrections such that $\mathbf{H}_{UQ}^\dagger = \mathbf{H}_{UQ}$, $\mathbf{H}_{NQ}^\dagger = -\mathbf{H}_{NQ}$, and

$$\begin{aligned}\mathcal{H}_{UC} &= -\mathcal{H}_D - \mathcal{H}_O - \mathcal{H}_{\rm vac.} + \frac{1}{m_P^2}\nabla S\cdot\mathbf{A} + \frac{\hbar}{m_P^2}\mathbf{A}\cdot\mathbf{A}_D - \frac{1}{2m_P^2}\langle\Phi|(i\hbar\nabla)^2|\Phi\rangle, \\ \mathcal{H}_{NC} &= -i\frac{\hbar}{m_P^2}\mathbf{A}\cdot\frac{\nabla F}{F} \end{aligned} \tag{12}$$

are c-numbers such that $\mathcal{H}_{UC}^* = \mathcal{H}_{UC}$, $\mathcal{H}_{NC}^* = -\mathcal{H}_{NC}$. Most of the c-numbers contribute physically uninteresting phase factors and will not be considered further. We briefly comment on the unitarity of quantum field. There are two terms possibly violating the unitarity, $\mathbf{H}_{NQ}$ and $\mathcal{H}_{NC}$. However, they are contributing the phase factors

$$\exp\Big(\frac{1}{i\hbar}\int\langle\Phi|\mathbf{H}_{NQ}|\Phi\rangle\Big) = \exp\Big(\frac{1}{m_P^2}\int\frac{\nabla F}{F}\cdot\mathbf{A}\Big) \tag{13}$$

and

$$\exp\Big(\frac{1}{i\hbar}\int\mathcal{H}_{NC}\Big) = \exp\Big(-\frac{1}{m_P^2}\int\frac{\nabla F}{F}\cdot\mathbf{A}\Big). \tag{14}$$

Therefore, they cancel each other, and Eq. (10) preserves the unitarity as shown in Refs. [7,8].

We shall now work with the semiclassical quantum gravity at the order of $\mathcal{O}(\hbar)$. Recollecting that $\mathbf{A}$ is of the order of $\mathcal{O}(\hbar)$ and the terms in the second square bracket in Eq. (7) and $\mathbf{H}_{UQ}$, $\mathbf{H}_{NQ}$ in Eq. (10) are all of the order of $\mathcal{O}(\hbar^2)$, one obtains the equations for semiclassical quantum gravity to the order of $\mathcal{O}(\hbar)$:

$$\frac{1}{2m_P^2}\big(\nabla S_{(0)}\big)^2 - \frac{1}{m_P^2}\mathbf{A}_{(0),O}\cdot\nabla S_{(0)} - m_P^2 V_{\rm ren.} + \mathcal{H}_{(0),D} + \mathcal{H}_{(0),O} = 0, \tag{15}$$

$$i\hbar\frac{\delta}{\delta\tau}|\Phi_{(0)}(\tau)\rangle = \mathbf{H}|\Phi_{(0)}(\tau)\rangle. \tag{16}$$

We make use of the well-known fact for a time-dependent quantum system that when the basis of the exact quantum states of Eq. (16) are chosen, the off-diagonal elements of the gauge potential are the same as those of the Hamiltonian [10]

$$\Big(\mathcal{H}_{(0),O}\Big)_{kn} = \langle\Phi_{(0),k}| : \mathbf{H} : |\Phi_{(0),n}\rangle = \langle\Phi_{(0),k}| i\hbar \frac{\delta}{\delta\tau} |\Phi_{(0),n}\rangle = \frac{1}{m_P^2}\Big(\mathbf{A}_{(0),O}\Big)_{kn}\cdot\nabla. \tag{17}$$

So these off-diagonal elements cancel among themselves. Equation (15) becomes the time-time component of the semiclassical Einstein equation in the form of Einstein-Hamilton-Jacobi equation

$$\frac{1}{2m_P^2}\Big(\nabla S_{(0)}\Big)^2 - m_P^2 V_{\text{ren.}} + \langle\langle\Phi_{(0)}| : \mathbf{H} : |\Phi_{(0)}\rangle\rangle = 0, \tag{18}$$

where $\mathcal{H}_{(0),D}$ is denoted by

$$\langle\langle\Phi_{(0)}| : \mathbf{H} : |\Phi_{(0)}\rangle\rangle \equiv \mathcal{H}_{(0),D} = \sum_{n\in\mathcal{S}} c_n^* c_n \langle\Phi_{(0),n}| : \mathbf{H} : |\Phi_{(0),n}\rangle. \tag{19}$$

Note that for a positive definite $\mathbf{H}$ each term in Eq. (18) takes positive value except for the trivial case of vacuum state. So the semiclassical quantum gravity allows only the positive energy density. Furthermore, for any two exclusive sets $\mathcal{S}_1$ and $\mathcal{S}_2$ such that $\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset$, superposition principle, which applies to classical gravity, also holds true for the quantum energy density:

$$\sum_{n\in\mathcal{S}_1\cup\mathcal{S}_2} \langle\langle\Phi_{(0)}| : \mathbf{H} : |\Phi_{(0)}\rangle\rangle = \sum_{n\in\mathcal{S}_1} \langle\langle\Phi_{(0)}| : \mathbf{H} : |\Phi_{(0)}\rangle\rangle + \sum_{n\in\mathcal{S}_2} \langle\langle\Phi_{(0)}| : \mathbf{H} : |\Phi_{(0)}\rangle\rangle. \tag{20}$$

This is to be compared with that of the traditional approach

$$\langle\Phi_{(0)}| : \mathbf{H} : |\Phi_{(0)}\rangle = \sum_{k,n\in\mathcal{S}} c_k^* c_n \langle\Phi_{(0),k}| : \mathbf{H} : |\Phi_{(0),n}\rangle, \tag{21}$$

where quantum interference among $k \neq n$ predominates in Eq. (21).

In order to illustrate the formalism developed so far, we shall consider a simple cosmological model with a minimal scalar field. Let us consider the minimal massive scalar field in a non-static Friedmann-Robertson-Walker universe with the metric

$$ds^2 = -N^2 dt^2 + a^2(t) d\Omega_3^2. \tag{22}$$

The corresponding Wheeler-DeWitt equation takes the form

$$\left[\frac{2\pi\hbar^2}{3m_P^2 a}\frac{\partial^2}{\partial a^2} - \frac{3m_P^2}{8\pi} ka + \Lambda a^3 + \frac{1}{2a^3}\hat{\pi}_\phi^2 + \frac{m^2 a^3}{2}\hat{\phi}^2\right]\Psi(a,\phi) = 0, \tag{23}$$

where $k = 1, 0, -1$ for a closed, flat and open universe, respectively, and Λ is the cosmological constant, and $\pi_\phi = a^3\dot{\phi}$. From Eq. (23) we derive the semiclassical Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} + \Lambda_{\text{ren.}} = \frac{8\pi}{3m_P^2 a^3} \langle\langle\Phi_{(0)}(\tau)| : \mathbf{H} : |\Phi_{(0)}(\tau)\rangle\rangle. \tag{24}$$

The cosmological time $\frac{\partial}{\partial\tau} = -\frac{4\pi}{3m_P^2 a}\frac{\partial S(a)}{\partial a}\frac{\partial}{\partial a}$ is identified with the comoving time t and no distinction will be made between τ and t. The Hamiltonian for the scalar field can decomposed into Fourier-modes [1]:

$$\mathbf{H} = \sum_\alpha \left[\frac{1}{2a^3}\hat{\pi}_\alpha^2 + \frac{a^3}{2}\Big(m^2 + \frac{\omega_\alpha^2}{a^2}\Big)\hat{\phi}_\alpha^2\right], \tag{25}$$

where ω_α^2 denote the eigenvalues of $-\nabla^2$. The exact quantum states for the Schrödinger equation are the number states, up to time-dependent phase factors, constructed from the annihilation and creation operators [8]

$$\begin{aligned} \hat{b}_\alpha(\tau) &= i\Big(\varphi_\alpha^*(\tau)\hat{\pi}_\alpha - a^3(\tau)\dot{\varphi}_\alpha^*(\tau)\hat{\phi}_\alpha\Big), \\ \hat{b}_\alpha^\dagger(\tau) &= -i\Big(\varphi_\alpha(\tau)\hat{\pi}_\alpha - a^3(\tau)\dot{\varphi}_\alpha(\tau)\hat{\phi}_\alpha\Big), \end{aligned} \tag{26}$$

where each mode satisfies the corresponding classical equation of motion

$$\ddot{\varphi}_\alpha + 3\frac{\dot{a}}{a}\dot{\varphi}_\alpha + \Big(m^2 + \frac{\omega_\alpha^2}{a^2}\Big)\varphi_\alpha = 0. \tag{27}$$

These operators are chosen to satisfy the commutation relations $[\hat{b}_\alpha, \hat{b}_\beta^\dagger] = \delta_{\alpha,\beta}$. Then the Hamiltonian has the oscillator representation

$$\mathbf{H} = \sum_\alpha C_\alpha(\hat{b}_\alpha^\dagger \hat{b}_\alpha + \hat{b}_\alpha \hat{b}_\alpha^\dagger) + D_\alpha \hat{b}_\alpha^2 + D_\alpha^* \hat{b}_\alpha^{\dagger 2}, \tag{28}$$

where

$$\begin{aligned} C_\alpha &= \frac{\hbar^2 a^3}{2}\Bigg[\dot{\varphi}_\alpha^* \dot{\varphi}_\alpha + \Big(m^2 + \frac{\omega_\alpha^2}{a^2}\Big)\varphi_\alpha^* \varphi_\alpha\Bigg], \\ D_\alpha &= \frac{\hbar^2 a^3}{2}\Bigg[\dot{\varphi}_\alpha^2 + \Big(m^2 + \frac{\omega_\alpha^2}{a^2}\Big)\varphi_\alpha^2\Bigg]. \end{aligned} \tag{29}$$

Following Ref. [8], we obtain the induced gauge potential

$$\mathbf{A} = \sum_\alpha 2C_\alpha \hat{b}_\alpha^\dagger \hat{b}_\alpha + D_\alpha \hat{b}_\alpha^2 + D_\alpha^* \hat{b}_\alpha^{\dagger 2}. \tag{30}$$

One thus sees that the off-diagonal elements of the Hamiltonian in Eq. (28) cancel exactly those of the gauge potential in Eq. (30). The Fock space constructed above can also be applied to a minimal massless scalar field except for the zero-mode.

We now wish to show that quantum interference may lead to negative energy density in the traditional approach and the loss of quantum coherence always leads to positive energy density in the semiclassical quantum gravity approach. First, let us consider the quantum state superposed of two numbers states $|n_\alpha, \tau\rangle$ and $|n_\alpha + 2, \tau\rangle$. The reason for choosing these quantum states is that particles are created or annihilated by pairs for the minimal massive scalar field in the curved spacetime. From the inequality $\left(\frac{D_\alpha + D_\alpha^*}{C_\alpha}\right)^2 < 4$, one can find the unique state

$$|\Phi_\alpha(\tau)\rangle = \frac{1}{\sqrt{1+\epsilon_\alpha^2}}\Big[|0,\tau\rangle + \epsilon_\alpha |2,\tau\rangle\Big], \tag{31}$$

that makes the normal ordered Hamiltonian

$$\langle \Phi_\alpha(\tau)| : \mathbf{H}_\alpha : |\Phi_\alpha(\tau)\rangle = \frac{1}{1+\epsilon_\alpha^2}\Big[4\epsilon_\alpha^2 C_\alpha + \sqrt{2}\epsilon_\alpha (D_\alpha + D_\alpha^*)\Big], \tag{32}$$

have negative energy density when

$$0 > \epsilon_\alpha > -\frac{1}{2\sqrt{2}}\Big(\frac{D_\alpha + D_\alpha^*}{C_\alpha}\Big), \quad \text{or} - \frac{1}{2\sqrt{2}}\Big(\frac{D_\alpha + D_\alpha^*}{C_\alpha}\Big) > \epsilon_\alpha > 0, \tag{33}$$

depending on the signs of $\left(\frac{D_\alpha + D_\alpha^*}{C_\alpha}\right)$. We have thus shown that the minimal massive scalar field can have the negative energy density [3]. We compare this with the energy density in the semiclassical quantum gravity

$$\langle\langle \Phi_\alpha(\tau)| : \mathbf{H}_\alpha : |\Phi_\alpha(\tau)\rangle\rangle = \frac{4\epsilon_\alpha^2}{1+\epsilon_\alpha^2} C_\alpha, \tag{34}$$

which is obviously positive definite.

Next, we show how classical matter Hamiltonian emerges from the semiclassical quantum gravity. It is well-known that coherent states of a quantum system have classical features. Let us consider the coherent states for each mode

$$|v_\alpha, \tau\rangle = e^{-\frac{|v_\alpha|^2}{2}} \sum_{n_\alpha = 0}^\infty \frac{v_\alpha^n}{\sqrt{n_\alpha!}} |n_\alpha, \tau\rangle. \tag{35}$$

It is straightforward to show

$$\langle\langle v_\alpha(\tau)| : \mathbf{H}_\alpha : |v_\alpha(\tau)\rangle\rangle = \frac{\hbar^2 a^3}{2} 2(v_\alpha^* v_\alpha) \left[\dot{\varphi}_\alpha^* \dot{\varphi}_\alpha + \Big(m^2 + \frac{\omega_\alpha^2}{a^2} \Big) \varphi_\alpha^* \varphi_\alpha \right]. \tag{36}$$

Note that Eq. (36) is $2(v_\alpha^* v_\alpha)$ times the vacuum expectation value. Each φ_α satisfies the corresponding classical equation of motion (27). By defining classical modes

$$\varphi_{\alpha,c} = \hbar \sqrt{2(v_\alpha^* v_\alpha)} \varphi_\alpha, \tag{37}$$

we obtain the classical Hamiltonian of a complex field $\pi_{\phi_c} = a^3 \dot{\phi}_c$:

$$\langle\langle v| : \mathbf{H} : |v\rangle\rangle = \frac{1}{2a^3} \pi_{\phi_c}^* \pi_{\phi_c} + \frac{m^2 a^3}{2} \phi_c^* \phi_c. \tag{38}$$

The ϕ_c satisfies the same classical equation for the real scalar field ϕ.

In summary, within the context of the semiclassical quantum gravity derived from the Wheeler-DeWitt equation, we have shown that superposition principle holds true for any exclusive set of quantum states due to the loss of quantum coherence and the energy density always takes positive value. Furthermore, through the study of a minimal massive scalar field in a non-static FRW spacetime it has been proved that the loss of quantum coherence makes the energy density take positive values only, which may take negative values due to the quantum coherence in the traditional approach. It should, however, be remarked that the induced gauge potential vanishes for a static spacetime and the coherence of the matter field recovers even in the semiclassical quantum gravity. The loss of quantum coherence (decoherence) is not due to the interaction of the matter field with an environment but entirely due to the interaction of the matter field with the gravity. We have also recovered the classical Hamiltonian of a complex field from the coherent state in the semiclassical quantum gravity. The result would have some cosmological implications, since any matter field in the Universe that changes the spacetime significantly should always provide positive energy density but certain gravity phenomena require negative energy density. It would also be interesting to test any possible modification to the quantum physics in the laboratory scale due to the ontological influence of the quantum cosmology.

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